Lesson 4-7: Using Corresponding Parts of Congruent Triangles

Putting it all together

Today we are going to pull together all that we've learned about triangles and apply it in more complicated situations. While we can find many examples of triangles in the "real world" many are a bit more complicated than the diagrams we've been working with. Many times the triangles are arranged in an overlapped manner.   In these cases it can be difficult to visualize how the triangles relate to each other and figure out how to determine congruence.

Today we will learn how to work with these types of triangle arrangements. In chapter 6 we will use these skills when we work with quadrilateral congruence.

Pull 'em apart

Here are some tips for working with diagrams with overlapped triangles:

1. Often the best thing to do is to redraw it with the triangles separated and labeled. When you do this it is much easier to identify corresponding and congruent parts.

2. A common side or angle is congruent to itself by the reflexive POC.

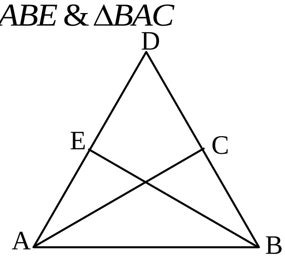
3. Sometimes you can prove one pair of triangles congruent and then use CPCTC to prove another pair congruent.

Consider the following examples.

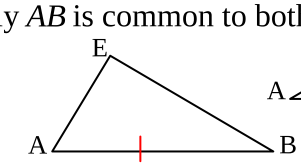
Examples – Pg 227

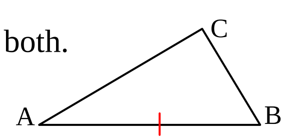
Separate & redraw. Identify any common angles or sides.

7. $\Delta ABE \;\&\; \Delta BAC$

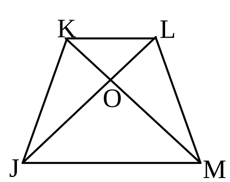


Only $\overline{AB}$ is common to both.

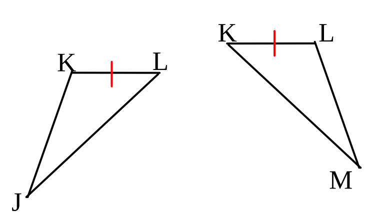




8. $\Delta JKL \;\&\; \Delta MLK$



Only $\overline{KL}$ is common to both.



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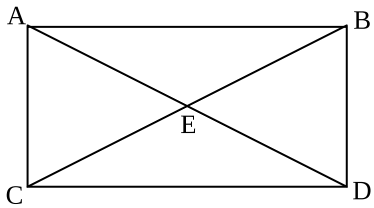
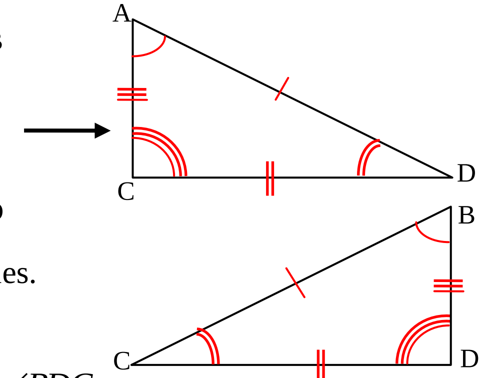
Example – Pg 225, Check Understanding 2

Plan and write a proof.

Given: $\Delta ACD \cong \Delta BDC$
Prove: $\overline{CE} \cong \overline{DE}$

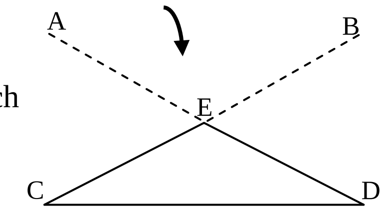




Plan: 1) Separate, redraw & label the congruent triangles.

2) Identify corresponding (and congruent) parts:
$\angle CAD \cong \angle DBC$, $\angle ADC \cong \angle BCD$, $\angle ACD \cong \angle BDC$,
$\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$, $\overline{CD} \cong \overline{CD}$

3) Next notice $\overline{CE}$ & $\overline{DE}$ are sides of ΔCED …redraw and label ΔCED:



4) Notice $\angle C$ (of ΔCED) is part of $\angle BCD$ and that $\angle D$ (of ΔCED) is part of $\angle ADC$ so $\angle C \cong \angle D$ which makes ΔCED an isosceles triangle.

Proof:	$\Delta ACD \cong \Delta BDC$	Given
	$\angle ADC \cong \angle BCD$	CPCTC
	Pt E is on $\overline{AD}$ & $\overline{BC}$	Given
	$\overline{CE} \cong \overline{DE}$	If 2 $\angle$'s $\cong$ then opposite sides are $\cong$ (Theorem 4-4)

Important

This example shows how separating and redrawing is a very important skill. Most of us have a difficult time looking at the first quadrilateral diagram and seeing the three important triangles.

Important

This example also shows that once you have two congruent triangles, you can use CPCTC to make conclusions about a third triangle.

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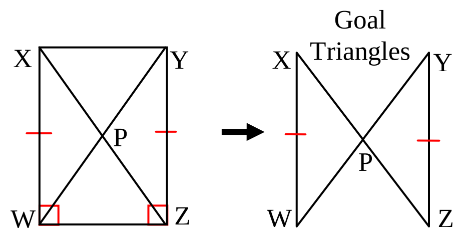
Example – not in the book

Given: $\overline{XW} \cong \overline{YZ}$

$\angle XWZ$ & $\angle YZW$ are right angles

Prove: $\Delta XPW \cong \Delta YPZ$



Plan: Separate, redraw & label:

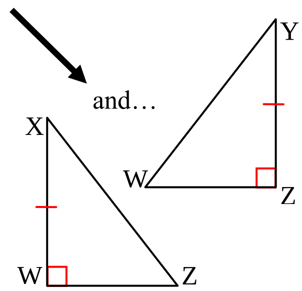
…goal triangles:

Notice the vertical angles $\angle XPW$ & $\angle YPZ$

…and other helpful triangle pairs:

Notice $\Delta XWZ \cong \Delta YZW$ (SAS: $\overline{WZ} \cong \overline{WZ}$)

We can use CPCTC to say $\angle WXZ \cong \angle WYZ$

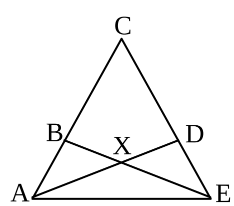


Proof:	$\overline{XW} \cong \overline{YZ}$	Given
	$\angle XWZ \cong \angle YZW$	All rt. $\angle$'s $\cong$
	$\overline{WZ} \cong \overline{WZ}$	Reflexive POC
	$\Delta XWZ \cong \Delta YZW$	SAS
	$\angle WXZ \cong \angle ZYW$	CPCTC
	$\angle XPW \cong \angle YPZ$	Vert. $\angle$'s $\cong$
	$\overline{XW} \cong \overline{YZ}$	Given
	$\Delta XPW \cong \Delta YPZ$	AAS

Example – not in the book

Given: $\overline{CA} \cong \overline{CE}$ & $\overline{BA} \cong \overline{DE}$

Prove: $\angle CBE \cong \angle CDA$



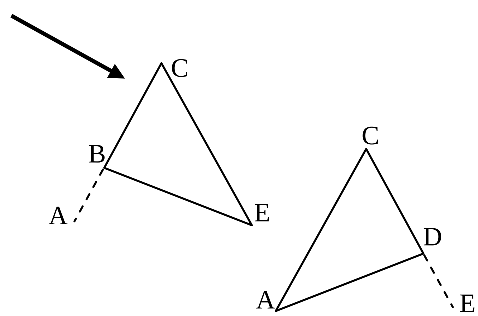
Plan: Separate, redraw & label:

…goal triangles:

Shared $\angle C$ & given $\overline{CA} \cong \overline{CE}$

Also notice $\overline{BA}$ is part of $\overline{CA}$

and $\overline{DE}$ is part of $\overline{CE}$



Proof:	$CA = CB + BA$	$\angle$ Add. Post.
	$CE = CD + DE$	$\angle$ Add. Post.
	$CA = CE$	Given
	$CB + BA = CD + DE$	Subst POE
	$BA = DE$	Given
	$CB + DE = CD + DE$	Subst POE
	$CB = CD, \overline{CB} \cong \overline{CD}$	Subtr POE
	$\angle C \cong \angle C$	Reflexive POC
	$\overline{CA} \cong \overline{CE}$	Given
	$\Delta CDA \cong \Delta CBE$	SAS
	$\angle CDA \cong \angle CBE$	CPCTC

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Assign homework

p. 226 #1-6, 9-14, 23, 24, 28, 34, 36, 38, 43, 45, 47
p. 232 #1-4